Published by Institute of Physics Publishing for SISSA/ISAS

Received: *January 16, 2005*
Accepted: *January 28, 2005*

Semiclassical string solutions on 1/2 BPS geometries

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Abstract: We study semiclassical string solutions on the 1/2 BPS geometry of type-IIB string theory characterized by concentric rings on the boundary plane. We consider both folded rotating strings carrying nonzero R-charge and circular pulsating strings. We find that unlike rotating strings, as far as circular pulsating strings are concerned, the dynamics remains qualitatively unchanged when the concentric rings replace $AdS_5 \times S^5$. Using the gravity dual we have also studied the Wilson loop of the corresponding gauge theory. The result is qualitatively the same as that in $AdS_5 \times S^5$ in the global coordinates where the corresponding gauge theory is defined on $S^3 \times R$. We show that there is a correction to $1/L$ leading order behavior of the potential between external objects.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

JHEP01(2005)050

Contents

1. Introduction

The original chain of reasonings leading to AdS/CFT correspondence relies on considering the near horizon geometry of type-IIB D3 brane supergravity solution and conjecturing a relation between string theory on this geometry and the field theory that lives on its boundary [1, 2, 3]. This geometry is $AdS_5 \times S^5$ in the Poincare coordinates and the dual theory is $\mathcal{N}= 4$ SU(N) SYM on R^4. These coordinates however do not cover all the AdS space and in order to do so one has to extend to the global coordinates which in turn amounts to changing the boundary to $S^3 \times R$ [3]. Therefore the global AdS is dual to CFT on $S^3 \times R$.[1]

This duality enables us to give geometric (gravitational) interpretation to different operators in the SYM by identifying the string excitations that they correspond to. Depending on energy, these excitations can range from point like field theory modes to brane configurations which can in principle cause geometric transition in $AdS_5 \times S^5$ due to back reaction.

Amongst all the operators in $\mathcal{N}= 4$ SYM on $S^3 \times R$ there is a certain class, known as 1/2 BPS, which is of special interest. These preserve half of the original supersymmetry and are specified by the condition $\Delta - J = 0$ where Δ is the conformal weight and J is a particular R symmetry charge of the operator. These operators have a free fermion field theory description and are characterized by the phase space of the fermions [5].

In order to find the geometric counterparts of this class of operators, the authors of [6] have established the general setting for obtaining the corresponding 1/2 BPS geometries. This is done by looking for those solutions of type IIB supergravity equations which have

[1] We note also that type-IIB string theory on the plane wave limit of the geometry is dual to a quantum mechanical theory [4].

$SO(4) \times SO(4) \times R$ isometry and which solve the killing spinor equations. Doing so, one picks from all the excitations in $AdS_5 \times S^5$ in the global coordinates, those which constitute its 1/2 BPS sector. It turns out that these symmetry requirements, plus regularity, are very restrictive such that the whole solution is determined by a single function which should satisfy a linear differential equation subject to certain boundary conditions on a 2-plane. The phase space of the underlying free fermion system is then identified with the different allowed configurations for this function on the 2-plane.

These families of solutions for constant axion and dilaton and zero three-form field strengths are given by

$$\begin{aligned} ds^2 &= -h^{-2}(dt^2 + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2 \,, \\ h^{-2} &= 2y \cosh G \,, \\ y\partial_y V_i &= \epsilon_{ij}\partial_j z \,, \qquad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z \,, \qquad z = \frac{1}{2}\tanh G \,, \end{aligned} \tag{1.1}$$

where $i, j = 1, 2$ and z satisfies the following equation

$$\partial_i \partial_j z + y\partial_y \left(\frac{\partial_y z}{y} \right) = 0 \,. \tag{1.2}$$

To get a nonsingular solution, z must obey the boundary condition $z = \pm 1/2$ at $y = 0$ on the 2-plane (x_1, x_2). The self dual five-form field strength is also nonzero (for details see [6]). One can now start with different allowed boundary conditions for z and obtain the corresponding solutions. For further studies in this direction see [7]–[12].

One would naturally like to study string excitations on each of the above backgrounds. String modes in different parts of the background can be studied by expanding string sigma model around classical configurations. These represent strings that are propagating in different parts of the space. The modes can in principle be non BPS and the deviation from BPS condition can be tuned by changing the classical charges of the configuration such as spin, angular momentum and etc. An important lesson from the semiclassical analysis of strings (see for example [13, 14, 15] and also [16] for reviews) is that when the charges are very large, a lot can be learned from the classical limit itself. This becomes an even better approximation when the number of charges is increased.

In a recent paper [12], rotating folded strings (first studied in [13]) have been considered in the 1/2 BPS geometry of type-IIB which is characterized by the concentring rings configuration on the (x_1, x_2) plane. This background is time independent and in certain limits can be thought of as a configuration of smeared S^5 giants and/or their AdS_5 duals. The corresponding metric in the polar coordinates is given by

$$\begin{aligned} ds^2 &= -h^{-2}(dt + V_r dr + V_\phi d\phi)^2 + h^2(dy^2 + dr^2 + r^2 d\phi^2) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2 \,, \\ h^{-2} &= 2y\cosh(G) \,, \qquad e^G = \sqrt{\frac{1+\tilde{z}}{-\tilde{z}}} \,, \end{aligned} \tag{1.3}$$

where

$$\tilde{z} = \frac{1}{2}\sum_{n=1}^{N}(-1)^{n+1}\left(\frac{r^2 - r_n^2 + y^2}{\sqrt{(r^2 + r_n^2 + y^2)^2 - 4r^2 r_n^2}} - 1 \right),$$

$$
\begin{aligned}
V_r &= 0\,, \\
V_\phi &= \frac{1}{2}\sum_{n=1}^{N}(-1)^n\left(\frac{r^2+r_n^2+y^2}{\sqrt{(r^2+r_n^2+y^2)^2-4r^2r_n^2}}-1\right).
\end{aligned} \tag{1.4}
$$

Here r_1 is the radius of the outermost circle, r_2 the next and so on. In the case of one radius, this is just one $AdS_5 \times S^5$. We will only consider the case where N is an odd number, therefore we will have a black disc in the middle of the configuration and $N-1$ rings. To fix our notation we may parameterize the two spheres as follows

$$
\begin{aligned}
d\Omega_3^2 &= d\theta_1^2 + \cos^2\theta_1(d\theta_2^2 + \cos^2\theta_2 d\theta^2)\,, \\
d\tilde{\Omega}_3^2 &= d\psi_1^2 + \cos^2\psi_1(d\psi_2^2 + \cos^2\psi_2 d\psi^2)\,.
\end{aligned} \tag{1.5}
$$

It was found in [12] that as compared to $AdS_5 \times S^5$, the concentric rings provide new physics for rotating strings. That is, a folded rotating string can have orbital angular momentum in S^3 in addition to the spin about its center of mass. Such orbiting strings had already been found in confining AdS backgrounds [17, 18] and in this sense the concentric rings show some sort of similarities with such backgrounds.

In the present work we continue the semiclassical analysis of strings in concentric rings. We start by generalizing the folded string of [12] to the case with nonzero angular momentum in $\tilde{S}^3$ and then focus on circular pulsating strings. These string configurations were first studied in $AdS_5 \times S^5$ in [19] and were then generalized to different situations [20]. We consider the concentric rings as a deformation of $AdS_5 \times S^5$ by taking the outermost radius to be much larger than the rest. This can be thought of as a configuration of a number of giants which are located near the pole of S^5 and close to one another and smeared in the polar coordinate of S^5.

We use Bohr-Sommerfeld analysis to find the energy levels of the pulsating string in terms of the level quantum number when the energy is very large. Our results show that unlike the rotating strings, pulsating ones experience no new physics and the dynamics is qualitatively that in the $AdS_5 \times S^5$ background. The rings affect the dynamics slightly only when the radius of the string becomes comparable to their radii.

The paper is organized as follows. In section 2, we will study rotating and folded closed strings in both long and short string limits on 1/2 BPS geometry (1.3) where the relation between their energy and spin is obtained. We will also consider another background which could be found by taking the Penrose limit of the concentric rings configuration. In section 3, we shall study the circular pulsating strings on this background that its energy relation is concluded. In section 4 we will study Wilson loop of the corresponding dual theory using open strings on this background. The last section is devoted to conclusions.

2. Rotating and spinning folded strings

In this section we shall study semiclassical closed strings in the 1/2 BPS geometry (1.3) which is extended in the y direction while rotating in both 3-spheres. Using our notation the corresponding closed string configuration is given by

$$
t = \kappa\tau\,, \qquad \theta = \omega\tau\,, \qquad \psi = \nu\tau\,, \qquad y = y(\sigma)\,, \tag{2.1}
$$

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and all other coordinates are set to zero. This closed string configuration has recently been studied in [12] where the authors have found the dependence of the energy of the string on the spin which represents the string's rotation in the first sphere of the solution (1.3). Here we will study a more general case where the string has nonzero angular momentum in both spheres.

The bosonic part of the GS superstring action for this closed string configuration on the background (1.3) is

$$S = \frac{1}{4\pi}\int d^2\sigma\,\left(h^{-2}\kappa^2 + h^2 y'^2 - \omega^2 y e^G - \nu^2 y e^{-G}\right). \tag{2.2}$$

For generic values of ω and ν this ansatz describes a classical string which is stretched and folded along y and rotates in the θ and ψ directions. The corresponding conserved charges of the system are

$$E = \frac{\kappa}{2\pi}\int_0^{2\pi} d\sigma\, 2y\cosh(G)\,, \qquad S = \frac{\omega}{2\pi}\int_0^{2\pi} d\sigma\, y e^G\,, \qquad J = \frac{\nu}{2\pi}\int_0^{2\pi} d\sigma\, y e^{-G}\,. \tag{2.3}$$

Now the aim is to find the dependence of energy E on S and J. To find this one may use the equation of motion derived from this action. We note however that it is useful to work with the first integral of the equation of motion which in this case is the Virasoro constraint

$$y'^2 = y^2\left[(\omega^2-\kappa^2)(1+e^{2G}) - (\kappa^2-\nu^2)(1+e^{-2G})\right]. \tag{2.4}$$

Using the expression for G we arrive at

$$y'^2 = y^2(\kappa^2-\nu^2)\left(\frac{1}{1+\tilde{z}} + \eta\,\frac{1}{\tilde{z}}\right), \tag{2.5}$$

where $\eta = \frac{\omega^2-\kappa^2}{\kappa^2-\nu^2}$. In terms of this parameter the turning points of the string along the y direction is given by $\frac{\tilde{z}_0}{1+\tilde{z}_0} = \eta$. Therefore in the simplest one folded case the interval $0 \le \sigma \le 2\pi$ is split into 4 segments in which for $0 \le \sigma \le \frac{\pi}{2}$ the function $y(\sigma)$ increases from zero to its maximal value given by z_0.

Using the definition of the energy, E, spin S and the angular momentum J of the string one sees that

$$E = \frac{\kappa}{\nu}J + \frac{\kappa}{\omega}S\,, \tag{2.6}$$

which together with the Virasoro constraint could be used to determine the dependence of E on S and J. Following [13] we shall study the limits of short string ($\eta \to \infty$) and long string ($\eta \to 0$), separately.

2.1 Short strings

The short string limit corresponds to the case where $\eta \to \infty$. In this case one may expand $\tilde{z}$ around $y = 0$

$$\tilde{z} = \sum_{n=1}^{N}(-1)^n \frac{r_n^2}{r_n^2+y^2} \sim -1 + f_0 y^2 - f_1 y^4\,, \tag{2.7}$$

where

$$f_0 = \sum_{n=1}^{N} (-1)^{n+1} \frac{1}{r_n^2} \,, \qquad f_1 = \sum_{n=1}^{N} (-1)^{n+1} \frac{1}{r_n^4} \,. \tag{2.8}$$

By making use of this expression one finds

$$y'^2 = \frac{\kappa^2 - \nu^2}{f_0} - (\kappa^2 - \nu^2) \left(\eta - \frac{f_1}{f_0^2} \right) y^2 + \mathcal{O}(y^4) \,. \tag{2.9}$$

The condition for having a singly folded string with radius $y_0 \ll 1$ is

$$(\kappa^2 - \nu^2) \left(\eta - \frac{f_1}{f_0^2} \right) = 1 \,, \qquad y_0^2 = \frac{\kappa^2 - \nu^2}{f_0} \,, \tag{2.10}$$

and therefore in leading order we get

$$\kappa^2 - \nu^2 \sim \frac{1}{\eta} \,, \qquad \omega^2 = \nu^2 + 1 + \frac{1}{\eta} \,. \tag{2.11}$$

On the other hand we have

$$y e^{G} \sim y^2 \sqrt{f_0} \left[1 + \frac{f_0}{2} \left(1 - \frac{f_1}{f_0^2} \right) y^2 \right] , \qquad y e^{-G} \sim \frac{1}{\sqrt{f_0}} \left[1 - \frac{f_0}{2} \left(1 - \frac{f_1}{f_0^2} \right) y^2 \right] , \tag{2.12}$$

which can be used to obtain the spin and angular momentum up to $\mathcal{O}(y^n)$ as follows

$$\begin{aligned} S &\approx \frac{\omega}{2\pi} \int_0^{2\pi} d\sigma \, \frac{\kappa^2 - \nu^2}{\sqrt{f_0}} \, \sin^2 \sigma \left[1 + \frac{\kappa^2 - \nu^2}{2} \left(1 - \frac{f_1}{f_0^2} \right) \sin^2 \sigma \right] , \\ J &\approx \frac{\nu}{2\pi} \int_0^{2\pi} d\sigma \frac{1}{\sqrt{f_0}} \left[1 - \frac{\kappa^2 - \nu^2}{2} \left(1 - \frac{f_1}{f_0^2} \right) \sin^2 \sigma \right] . \end{aligned} \tag{2.13}$$

Here we have used the fact that $y^2 = \frac{\kappa^2 - \nu^2}{f_0} \sin^2 \sigma$. By making use of $\kappa^2 - \nu^2 \sim \frac{1}{\eta}$ one finds

$$\begin{aligned} S &\sim \frac{\omega}{2\sqrt{f_0}} \frac{1}{\eta} \left[1 + \frac{3}{8} \left(1 - \frac{f_1}{f_0^2} \right) \frac{1}{\eta} \right] , \\ J &\sim \frac{\nu}{\sqrt{f_0}} \left[1 - \frac{1}{4} \left(1 - \frac{f_1}{f_0^2} \right) \frac{1}{\eta} \right] . \end{aligned} \tag{2.14}$$

Altogether in leading order we arrive at

$$\nu = \sqrt{f_0} J \,, \qquad \kappa^2 \sim f_0 J^2 + \frac{2\sqrt{f_0} S}{\sqrt{f_0 J^2 + 1}} \,, \qquad \omega^2 \sim 1 + f_0 J^2 + \frac{2\sqrt{f_0} S}{\sqrt{f_0 J^2 + 1}} \,. \tag{2.15}$$

Plugging these into (2.6) one gets

$$E \approx \sqrt{J^2 + \frac{2S}{\sqrt{f_0^2 J^2 + f_0}}} \left(1 + \frac{S}{\sqrt{J^2 + \frac{2S}{\sqrt{f_0^2 J^2 + f_0}} + \frac{1}{f_0}}} \right) . \tag{2.16}$$

For the situation where both J and S are small one finds

$$E^2 \approx J^2 + \frac{2S}{\sqrt{f_0}} \,, \tag{2.17}$$

which actually represents the Reggae trajectories in the flat space. On the other hand for the case where $J \ll S$ we get

$$E \approx \frac{\sqrt{2S}}{f_0^{1/4}} + \frac{J^2}{2f_0^{1/4}\sqrt{2S}}, \tag{2.18}$$

which, for the limit of $J \gg 2S$, it leads to

$$E \approx J + S + \frac{S}{2f_0 J^2}. \tag{2.19}$$

We note that these are exactly the same expressions found in [14], where the authors have studied the same string configuration as (2.1) on the $AdS_5 \times S^5$, if we identify λ with $1/f_0$ where λ is radius of the AdS space.

2.2 Long strings

In the long string limit we have $\eta \to 0$ where the maximal value of y_0 is large. In this case one may expand $\tilde{z}$ for large y

$$z = \sum_{n=1}^{N} (-1)^n \frac{r_n^2}{r_n^2 + y^2} \approx -\frac{g_0}{y^2} + \frac{g_1}{y^4}, \tag{2.20}$$

where

$$g_0 = \sum_{n=1}^{N} (-1)^{n+1} r_n^2, \qquad g_1 = \sum_{n=1}^{N} (-1)^{n+1} r_n^4, \tag{2.21}$$

therefore one finds

$$y'^2 = y^2(\kappa^2 - \nu^2)\left(1 - \frac{\eta}{g_0} y^2\right). \tag{2.22}$$

We note that for $0 < \sigma < \frac{\pi}{2}$ the function $y(\sigma)$ increases from zero to its maximal value y_0 given by $y_0 = \sqrt{g_0/\eta}$, so

$$2\pi = \int_0^{2\pi} d\sigma = \frac{4}{\sqrt{\kappa^2 - \nu^2}} \int_0^{y_0} \frac{dy}{y\sqrt{1 - \frac{\eta}{g_0} y^2}}. \tag{2.23}$$

Therefore we get

$$\kappa^2 \sim \nu^2 + \frac{1}{\pi^2} \ln^2 \frac{g_0}{\eta}, \qquad \omega^2 \sim \nu^2 + \frac{1}{\pi^2}(1 + \eta) \ln^2 \frac{g_0}{\eta}. \tag{2.24}$$

On the other hand we find

$$\begin{aligned} ye^G &\sim \frac{y^2}{\sqrt{g_0}} \left(1 - \frac{g_0}{2}\left(1 - \frac{g_1}{g_0^2}\right)\frac{1}{y^2}\right), \\ ye^{-G} &\sim \sqrt{g_0}\left(1 + \frac{g_0}{2}\left(1 - \frac{g_1}{g_0^2}\right)\frac{1}{y^2}\right). \end{aligned} \tag{2.25}$$

Plugging these in the expressions of S and J and using the equation (2.22) we arrive at

$$S \approx \frac{\omega\sqrt{g_0}}{\eta \ln \frac{g_0}{\eta}}, \qquad J = \sqrt{g_0}\,\nu. \tag{2.26}$$

For the case of $\nu \ll \ln \frac{g_0}{\eta}$ this results $\sqrt{g_0}/\eta \sim \pi S/2$ and by making use of the relation (2.6) we find

$$E \approx S + \frac{\sqrt{g_0}}{\pi} \ln \frac{S}{\sqrt{g_0}} + \frac{\pi J^2}{2\sqrt{g_0} \ln \frac{S}{\sqrt{g_0}}} . \tag{2.27}$$

On the other hand in the opposite limit where $\nu \gg \ln \frac{g_0}{\eta}$ we get

$$E \approx S + J + \frac{\sqrt{g_0}}{2\pi^2 J} \ln^2 \frac{S}{J} . \tag{2.28}$$

We note that in comparison with the AdS case studied in [14] we get the same expressions if we identify the AdS radius with $\sqrt{g_0}$.

So far it seems that the folded closed strings (2.1) on the 1/2 BPS geometry (1.3) qualitatively feel the same physics as the AdS background as far as the large and small energy regimes are concerned. We note however that in the intermediate scale the new physics might appear. In fact it was shown [12] that this is the case at least for the rotating folded closed strings. This can be understood as follows.

From the first integral of motion we see that the condition for having a folded string is equivalent to the condition for having two turning points for a one-dimensional motion in the effective potential [12]

$$V(y) = -\frac{1}{\tilde{z}} = \frac{-1}{\sum (1)^{n+1} \frac{r_n^2}{r_n^2 + y^2}} . \tag{2.29}$$

This potential has several minima and therefore going from large scale into the intermediate scale the string can split into smaller folded orbiting strings which are located around these new minima.

There is an other interesting situation one may have because of these new minima. In fact one can consider a folded closed string localized around one of the minima with large J charge corresponding to the high angular momentum in the ψ direction. In this situation, if we had considered AdS case, we would have got the plane wave background for the quadratic fluctuations around this classical solution. In this case we would also expect to get the plane wave solution for each minima. One might also suspect that cutting a small strip, say around $\phi = \pi/2$, in the boundary plane of the solution (1.3), could result in a new solution which is the superposition of the plane wave solutions we get from each minimum.

To be more precise, we consider the concentric rings background as a deformation of $AdS_5 \times S^5$ by demanding that $r_n - r_N \ll r_N$. This can be viewed as an $AdS_5 \times S^5$ with the radius $r_1(r_N)$ containing a number of giants (AdS giants) which are located close to one another and close to the equator of S^5, $\theta = 0$, smeared in the polar coordinate. With this assumption we expand the rings around the point $r = r_N$, $y = 0$ and $\phi = \pi/2$ by defining

$$r - r_N \equiv \frac{x_2}{r_N} , \qquad y \equiv \frac{w}{r_N} , \qquad r_n - r_N \equiv \frac{R_n}{r_N} , \qquad \phi - \frac{\pi}{2} \equiv -\frac{x_1}{r_N^2} . \tag{2.30}$$

Writing the concentric rings solution in terms of the above parameters, taking the limit of $r_N \to \infty$ and renaming w as y, will result in a background which is the superposition of a number of plane waves

$$
\begin{aligned}
z(x_2, y) &= \frac{1}{2}\sum_{n=1}^{N}(-1)^{n+1}\frac{x_2 - R_n}{\sqrt{(x_2 - r_n)^2 + y^2}} \qquad (R_N = 0)\,, \\
V_1 &= V_\phi\, \partial_{x_1}\phi = \frac{1}{2}\sum_{n=1}^{N}(-1)^{n+1}\frac{1}{\sqrt{(x_2 - R_n)^2 + y^2}}\,, \qquad V_2 = 0\,.
\end{aligned} \tag{2.31}
$$

This background can be found directly from the equations of motion by demanding the following boundary condition on the (x_1, x_2) plane [6]

$$
z(x_1', x_2', 0) = \frac{1}{2}\sum_{n=1}^{N}(-1)^{n+1} Sign(x_2' - R_n)\,, \tag{2.32}
$$

where we take N to be an odd number such that the solution becomes pp wave asymptotically. This boundary condition is shown by a number of horizontal black and white strips on the (x_1, x_2) plane bounded by black and white regions from below and above respectively and thus we call its corresponding solution the "*Zebra*" background. It is easy to show that this boundary condition will result in the solution (2.31).

3. Circular pulsating strings

We now study pulsating strings in the background (1.3). These string solutions were first studied in [19] on $AdS_5 \times S^5$ and generalizations to other backgrounds were given in [20]. Let us first briefly review the $AdS_5 \times S^5$ case. A pulsating string is defined through the following ansatz for the embedding coordinates

$$
t = \tau\,, \qquad \rho = \rho(\tau)\,, \qquad \theta = m\sigma\,, \tag{3.1}
$$

where ρ is the radial direction of AdS in the global coordinates and θ is a great circle of the S^3 contained in AdS_5. The rest of the coordinates are taken to be zero. Writing the Nambu-Goto action for this configuration, we arrive at a one dimensional quantum mechanical system with the hamiltonian

$$
H = \sqrt{\Pi^2 + m^2\lambda \tan^2\xi \sec^2\xi}\,, \tag{3.2}
$$

where λ is the 't Hooft coupling, $\xi = \sin^{-1}(\tanh\rho)$ and Π is the conjugate momentum of ξ. One can then define a potential

$$
V(\xi) = m^2 \frac{\lambda \tan^2\xi}{\cos^2\xi}\,, \tag{3.3}
$$

for the system with the hamiltonian H^2 and find the energy levels.

With this brief review we now turn to the problem of a pulsating string in (1.3). We consider the following ansatz

$$t = \tau\,, \qquad y = y(\tau)\,, \qquad \theta = m\sigma\,, \tag{3.4}$$

and the rest of coordinates are zero. The Nambu-Goto action for this configuration reads

$$S_{NG} = -\frac{m}{\alpha'}\int dt\, g(\xi)\sqrt{1-\dot{\xi}^2}\,, \tag{3.5}$$

where

$$\frac{d\xi}{dy} = h^2\,, \qquad g(\xi) = \frac{y}{\sqrt{-\tilde{z}}}\,. \tag{3.6}$$

We find the momentum and hamiltonian for the system as

$$\Pi = \frac{m}{\alpha'}\, g(\xi)\,\frac{\dot{\xi}}{\sqrt{1-\dot{\xi}^2}}\,, \qquad H = \sqrt{\Pi^2 + \left(\frac{m}{\alpha'}\right)^2 g(\xi)^2}\,. \tag{3.7}$$

Now one can consider H^2 as a one dimensional system with the potential

$$V(\xi) = \left(\frac{m}{\alpha'}\right)^2 g(\xi)^2\,. \tag{3.8}$$

To compare this potential with the one in (3.3), we define the variables ξ_n and the constants λ_n by

$$\cos^2\xi_n = \frac{r_n^2}{y^2+r_n^2}\,, \qquad \sin^2\xi_n = \frac{y^2}{y^2+r_n^2}\,, \qquad \lambda_n = \frac{r_n^2}{\alpha'^2}\,, \tag{3.9}$$

where it is clear that these variables are not independent, as they are all determined by y, and vary between zero and $\xi_n^{\max}$ which are determined by $y_{\max}$. One should also note that $\xi_1 < \xi_2 < \cdots < \xi_N$ for a given value of y. In terms of these parameters the potential can be brought to a form which can be easily compared with (3.3)

$$V = \frac{m^2}{N}\frac{\sum_{n=1}^N \lambda_n \tan^2\xi_n}{\sum_{n=1}^N \cos^2\xi_n}\,. \tag{3.10}$$

For $N = 1$, the above potential reduces to (3.3). The qualitative behavior of this potential is the same as that in the $AdS_5 \times S^5$ case; classically the circular string pulsates between a point and a circle whose radius is determined by the string's energy. So, as far as pulsating strings are concerned, no new physics emerges in this problem. This should be compared with the folded strings of the previous section for which the new exterma in the potential give rise to orbiting strings in addition to spinning ones.

We can now proceed to find the energy levels of our one dimensional quantum mechanical system with the potential given by (3.8). We are mainly interested in string states with a large quantum number which, in our problem, is the level number. Therefore we focus on large energy states for which the Bohr-Sommerfeld analysis is a good approximation. As the energy of the string increases, the $\xi_n^{\max}$ get closer to $\pi/2$. Since y is a radial coordinate

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we will symmetrize the potential around $y = 0$ by allowing the ξ_n to range between $-\pi/2$ and $\pi/2$ and consider only the even wave functions

$$\left(2n+\frac{1}{2}\right)\pi \approx \int_{-\xi_0}^{\xi_0} d\xi \sqrt{E^2 - \left(\frac{m}{\alpha'}\right)^2 g(\xi)^2}\,, \qquad n = 0,1,2,\dots\,, \tag{3.11}$$

where $\pm\xi_0$ are found from the equation

$$E = \left(\frac{m}{\alpha'}\right) g(\xi_0)\,. \tag{3.12}$$

To perform the integrations we consider the background (1.3) as a deformation of $AdS_5 \times S^5$ by assuming that the radius of the outermost circle is much larger than the rest

$$R \equiv r_1\,, \qquad \beta_n \equiv \frac{r_n}{R} \ll 1 \quad (1 < n \leq N)\,. \tag{3.13}$$

We define the large energy limit of the string by $B \equiv E(m\sqrt{\lambda_1})^{-1} \gg 1$. The limit of large B and small β means that the string spends most of its time in the region where $\xi_n\,(1 < n \leq N)$ have almost saturated their bound at $\pi/2$ and therefore it is as if the string is pulsating in $AdS_5 \times S^5$. The new features of the background (1.3) play a part in the dynamics only when the string has shrunk to a small circle. The radius of this circle becomes arbitrarily small as $\beta \to 0$ and the maximum radius of the pulsating string becomes arbitrarily large as $B \to \infty$. With these assumptions we can write the following expansions for h and V around their values in the $AdS_5 \times S^5$ background

$$\begin{aligned} h^2 &\approx \frac{1}{BR}\,\frac{1}{B^{-1}+x^2}\left[1+\frac{1}{2}\beta^2\,\frac{B^{-2}-x^4}{x^4}\right], \\ V &\approx E^2x^2(B^{-1}+x^2)\left[1+\beta^2\,\frac{B^{-1}+x^2}{x^2}\right], \end{aligned} \tag{3.14}$$

where we have defined

$$x = \frac{y}{R\sqrt{B}}\,, \qquad \beta^2 = \sum_{n=2}^{N}(-1)^n\beta_n^2 = 1 - \frac{g_0}{R^2}\,. \tag{3.15}$$

It is useful to write the integral in (3.11) as the sum of two integrals as following

$$\begin{aligned} \frac{2mR}{\alpha'}\sqrt{B}\Bigg\{ &\int_0^{x_0}\frac{dx}{B^{-1}+x^2}\left[1+\frac{1}{2}\beta^2\frac{B^{-2}-x^4}{x^4}\right] - \\ &-\int_0^{x_0}\frac{dx}{B^{-1}+x^2}\left[1+\frac{1}{2}\beta^2\,\frac{B^{-2}-x^4}{x^4}\right]\left[1-\sqrt{1-\frac{V}{E^2}}\right]\Bigg\}. \end{aligned} \tag{3.16}$$

The first integral is nothing but ξ_0 which can be found as

$$\xi_0 = BR\int_0^{x_0} dx\, h^2 \approx \tan^{-1}(\sqrt{B}x_0) + \frac{1}{2}\beta^2\,\frac{3Bx_0^2-1}{3B^{3/2}x_0^3}\,. \tag{3.17}$$

Noting that $x = 1/\sqrt{B}\tan\xi_1$, we find from the above relation that ξ_0 is approximately $\xi_1^{\max}$ (for small β) which is very close to $\pi/2$ (for large B). One can use (3.12) to find

$$\xi_0 \approx \frac{\pi}{2} - \frac{1}{\sqrt{B}}\left(1 - \frac{1}{4}\beta^2\right). \tag{3.18}$$

The second integral in (3.16) remains finite in the large B limit and is written as

$$\left(1 - \frac{1}{4}\beta^2\right)\int_0^1 \frac{du}{u^2}\left(1 - \sqrt{1-u^4}\right) = \left(1 - \frac{1}{4}\beta^2\right)\left(-1 + \frac{(2\pi)^{3/2}}{\Gamma(\frac{1}{4})^2}\right). \tag{3.19}$$

Therefore the large energy approximation takes the following form

$$\left(2n + \frac{1}{2}\right)\pi \approx E\pi - \left(1 - \frac{1}{4}\beta^2\right)\frac{4\pi(2\pi)^{1/2}}{\Gamma(\frac{1}{4})^2}m^{1/2}\lambda_N^{1/4}\sqrt{E}. \tag{3.20}$$

We can invert this relation and find

$$E \approx 2n + \left(1 - \frac{1}{4}\beta^2\right)\frac{8\pi^{1/2}}{\Gamma(\frac{1}{4})^2}\lambda_N^{1/4}\sqrt{mn}. \tag{3.21}$$

This is our final expression for the large energy states of a circular string, in terms of a large number n, when it pulsates in the background (1.3) which is considered as a deformation of $AdS_5 \times S^5$.

4. Wilson loop

In the previous sections we have studied semiclassical closed strings in the 1/2 BPS geometries. We note however that the open string configurations in a given gravity background could also be used to study the Wilson loop and thereby the potential between external objects in the corresponding dual field theory [21, 22].

In this section we shall study the open string solutions in the 1/2 BPS geometries. One may then identify this with the Wilosn loop in the dual gauge theory which is presumably living on the boundary with the topology of $S^3 \times R$.

To warm up we will first study the Wilson loop in $\mathcal{N} = 4$ SYM theory on $S^3 \times R$. This can be done using the corresponding supergravity description which in this case is $AdS_5 \times S^5$ in the global coordinates given by

$$\begin{aligned} ds^2 &= -R^2\left(1 + \frac{r^2}{R^2}\right)dt^2 + \frac{dr^2}{1 + \frac{r^2}{R^2}} + r^2 d\Omega_3^2 + R^2 d\Omega_5^2, \\ d\Omega_3^2 &= d\theta_1^2 + \cos^2\theta_1(d\theta_2^2 + \cos^2\theta_2\, d\theta^2). \end{aligned} \tag{4.1}$$

Here we have used a unit in which t is dimensionless.

Let us now consider the following open string configuration

$$t=\tau\,,\qquad \theta=\sigma\,,\qquad r=r(\sigma)\,,\qquad \theta_1=\theta_2=0\,. \tag{4.2}$$

The classical string action for the above string configuration is obtained as the following

$$S=\frac{R}{2\pi\alpha'}\int dt\,d\theta\,\sqrt{r'^2+r^2\left(1+\frac{r^2}{R^2}\right)}\,, \tag{4.3}$$

where prime denotes derivative with respect to θ. Since the action is θ independent, the corresponding hamiltonian is constant of motion leading to

$$\frac{\frac{r^4}{R^2}(1+\frac{R^2}{r^2})}{\sqrt{r'^2+\frac{r^4}{R^2}(1+\frac{R^2}{r^2})}}=\frac{r_0^2}{R}\sqrt{1+\frac{R^2}{r_0^2}}\,, \tag{4.4}$$

where r_0 is the point where $r'=0$.

Setting $y=r/r_0$ and $\epsilon=R/r_0$ one may find the distance between two external objects in the theory as follows

$$\frac{\theta}{2}=\frac{R}{r_0}\int_1^\infty\frac{dy}{y^2(1+\epsilon^2/y^2)^{1/2}\sqrt{y^4\frac{1+\epsilon^2/y^2}{1+\epsilon^2}-1}}\,. \tag{4.5}$$

The potential energy is given by

$$E=\frac{Rr_0}{2\pi\alpha'}\left[\int_1^\infty dy\left(\frac{y^2}{\sqrt{y^2-\frac{1+\epsilon^2}{1+\epsilon^2/y^2}}}-1\right)-1\right]. \tag{4.6}$$

The aim is now to eliminate r_0 between these expression to find E in terms of θ. In general it is difficult to do so. Nonetheless one may work in the limit where $\epsilon\ll 1$. In the limit of $\epsilon\to 0$ one would expect to get the same result as in the theory on the Minkowski space which is dual to the theory on AdS in Poincare patch. For small ϵ one then expects to get some corrections which could be due to short distance effects taking into account that in this case the theory is defined on a sphere.

In fact expanding the above expression in ϵ one gets

$$\begin{aligned}\frac{\theta}{2}&=\frac{R}{r_0}\int_1^\infty\frac{dy}{y^2\sqrt{y^4-1}}\left(1+\frac{y^4-y^2-1}{2y^2(1+y^2)}\epsilon^2+\cdots\right),\\ E&=\frac{Rr_0}{2\pi\alpha'}\int_1^\infty dy\left[\frac{y^2}{\sqrt{y^4-1}}\left(1+\frac{\epsilon^2}{2y^2(1+y^2)}-\frac{(4y^2+1)\epsilon^4}{8y^4(1+y^2)^2}+\cdots\right)-1\right]-\frac{Rr_0}{2\pi\alpha'}\,.\end{aligned} \tag{4.7}$$

Therefore in leading order we find

$$E\sim-\frac{R^2}{2\pi\alpha'}\,\frac{1}{\theta}\,(1+c_0\theta^2+c_1\theta^2\cdots)\,, \tag{4.8}$$

where c_i's are some numerical constants. As we see, besides the standard R^2/θ term we have some corrections which could be understood due to short distance effects. We note that corrections to the Wilson loop, we have here, might be related to those in [23] where the authors have studied the Wilson loop in gauge theory side when there is a cusp in the loop. There the authors have found corrections to the Wilson loop proportional to the cusp angle. One might then wonder that our corrections have the same origin under mapping to plane.[2]

Let us now consider the following open string configuration in the background (1.3)

$$t=\tau\,,\qquad \theta=\sigma\,,\qquad y=y(\theta)\,,\qquad r=0\,. \tag{4.9}$$

The corresponding classical action is given by

$$S=\frac{T}{2\pi\alpha'}\int d\theta\sqrt{y'^2+\frac{y^2}{-\tilde{z}}}\,. \tag{4.10}$$

To get an insight of what kind of physics one might get we shall consider the case where $r_0 \gg r_N$. Here r_0 is the turning point of string where y' is zero. In this limit one may expand $\tilde{z}$ for large y getting

$$S\sim\frac{T}{2\pi\alpha'}\int d\theta\sqrt{\left(\frac{dy}{d\theta}\right)^2+y^2\frac{g_1}{g_0^2}\left(1+\frac{g_0}{g_1}y^2\right)}\,. \tag{4.11}$$

Setting $\tilde{\theta}=\theta\frac{g_1^{1/2}}{g_0}$ one finds

$$S\sim\frac{T}{2\pi\alpha'}\int d\tilde{\theta}\sqrt{\left(\frac{dy}{d\tilde{\theta}}\right)^2+y^2\left(1+\frac{y^2}{R^2}\right)}\,, \tag{4.12}$$

where $R^2=g_1/g_0$. Using the result of the AdS space in the global coordinates one can now easily read the potential of the external objects in terms of their distance in θ direction. In particular at leading order one gets

$$E\sim-\frac{\sqrt{g_1}}{\theta}+\mathcal{O}(\theta)\,, \tag{4.13}$$

showing that the Wilson loop probes the background with the characteristic length $\sqrt{g_1}$.

5. Conclusion

In this paper we have studied the 1/2 BPS geometry of type-IIB which is characterized by concentric rings on the boundary 2-plane. The number of radii of the rings is taken to be odd such that the background is asymptotically $AdS_5\times S^5$. Our probe which explores this space time is a closed string that propagates in the background according to the classical equations of motion.

The first configuration we have considered is the one studied in [12]; a folded closed string extended in the radial direction of AdS, spinning around its center of mass and orbiting in S^3, and at the same time rotating in the $\tilde{S}^3$ with a maximal radius. The one

[2]We would like to thank Albion Lawrence for a discussion on this point.

dimensional potential governing the dynamics of these strings allows the radial coordinate of the center of mass to be different from zero and hence orbiting strings, in addition to spinning ones, appear in the problem. This should be compared to $AdS_5 \times S^5$ where only spinning strings were allowed. As stated before, orbiting configurations for strings are permitted in confining AdS backgrounds and in this sense, the concentric rings show some similarities with such backgrounds. In this work we extended the results of [12] to cases with nonzero $\tilde{S}^3$ angular momentum and found the energy dependence on the angular momenta for short and long strings centered around the origin for large values of the classical charges.

One could also consider folded configurations in the regions where $\tilde{S}^3$ has shrunk to a point with the angular momentum coming from rotations in the remaining angle of S^5. In the $AdS_5 \times S^5$ case, the string sigma model expansion around such a configuration, in the point like string limit, would lead to the pp wave background. In the present case one would reasonably expect that a similar analysis could yield a similar result. Our expectation is that for a shrunk $\tilde{S}^3$, the new exterma are still present in the potential and these will allow for several point like configurations such that the sigma model expansion around each point would result in a pp wave like limit.

Based on this expectation, we were able to find a limit of the rings background which is a superposition of a number of pp waves. For this purpose we considered the rings as a deformation of $AdS_5 \times S^5$ by taking very narrow rings at the edge of the droplet. This superposition of pp waves is itself a solution of the equations of motion which is characterized by a number of horizontal black and white strips on the boundary plane bounded by black and white regions from below and above respectively. We called this background "*Zebra*".

We also studied circular strings wound around the equator of S^3 and pulsating in the radial direction of AdS. The system reduces to a one dimensional quantum mechanical system for this configuration and we use Bohr-Sommerfeld analysis to find the energy levels of the system when the level number is large. We were able to do the calculations when the rings are considered as a deformation of $AdS_5 \times S^5$. This time the deformation is produced by taking very narrow rings close to the center of the droplet. The results show that unlike the folded strings, pulsating ones experience no new physics in this background as compared to $AdS_5 \times S^5$. Our end result is an expansion for the energy of the string in terms of the level (and winding) number and the deformation parameters.

We have also studied Wilson loop of the corresponding dual theory, using open strings on 1/2 BPS geometry (1.3) where we found some corrections which could be due to short distance effects, showing that the dual theory must be defined on a sphere with characteristic length $\sqrt{g_1}$.

Finally we note that, although the results we have found are qualitatively the same as those in $Ads_5 \times S^5$, the strings do see a new structure of the background. In fact as we have seen different strings probe this background with different parameters. For example in Wilson loop the parameter is given by $\sqrt{g_1}$, while for the rotating short and long strings it is given by $\sqrt{f_0}$ and $\sqrt{g_0}$, respectively.

JHEP01(2005)050

Acknowledgments

We would like to thank Mohsen Alishahiha for collaboration in the early stage of this work and useful comments and discussions. We would also like to thank Albion Lawrence and Soo-Jong Rey for useful discussions.

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